Particle production by Schwinger mechanism

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Non-perturbative particle production mechanism in heavy-ion collisions

- Treat gluon fields as classical electric fields.
- The electric field decays into particles by Schwinger mechanism.

The momentum distribution of created particles and its time-evolution are not fully understood.

We investigate

- Dynamical view of pair creation
Non-perturbative particle production mechanism in heavy-ion collisions

- Treat gluon fields as classical electric fields.
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The momentum distribution of created particles and its time-evolution are not fully understood.

Color glass condensate (Lappi, McLerran 2006)

Both electric and magnetic field exist

We investigate

- Dynamical view of pair creation
- Effects of a magnetic field
**Schwinger mechanism**  
Schwinger 1951  
Non-perturbative pair creation mechanism in an uniform and static classical electric field

**Vacuum persistence probability**
\[
|\langle 0, \text{out} | 0, \text{in} \rangle|^2 = \exp(-2 \text{Im } L) = \exp(-\int \omega d^4 x)
\]

**Pair creation probability per unit volume and unit time**
\[
w = \frac{2(eE)^2}{(2\pi)^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n\pi m^2}{eE}\right) \text{ (fermion)}
\]

No direct information about particle number time evolution
**Schwinger mechanism**  
Schwinger 1951  
Non-perturbative pair creation mechanism in an uniform and static classical electric field

**Vacuum persistence probability**

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\]

**Particle mean number**

\[
\langle 0, \text{in} | a_\mathbf{p}^\dagger(t) a_\mathbf{p}(t) | 0, \text{in} \rangle
\]

Direct information about particle number time evolution  

Need to define particle picture specifically
sudden-switch-on electric field

To get dynamical view of pair creation, we need to deal with non-steady electric fields.

In a static electric field, a distribution is also static.
Canonical quantization in a constant electric field

Klein-Gordon field and Dirac field interacting with the classical electric field $\mathbf{E} = (0, 0, E)$

$$[\left( \partial_\mu + i e A_\mu \right)^2 + m^2] \phi(x) = 0, \left[ \gamma^\mu (i \partial_\mu - e A_\mu) - m \right] \psi(x) = 0$$

In the case of non-Abelian electric field $\mathbf{E}^a = (0, 0, E)n^a$, gluon field and quark field obey the equations of the same form as in Abelian case, up to first order.

But in the case of gluon, magnetic moment causes a difficulty

**Quantization**

$$[\phi(t, \mathbf{x}), \pi(t, \mathbf{x'})] = i \delta^3(\mathbf{x} - \mathbf{x'})$$

Particle picture is established.

ambiguity in an external field
Instantaneous particle picture

\[ A_z = -Et \]

\[ +\phi^\text{in}_p(t) \]

\[ \mathbf{A} = 0 \]

\[ +\phi^\text{in}_p(t) = \frac{1}{\sqrt{2\omega_p}} e^{-i\omega t} \]

Positive frequency
Instantaneous particle picture

\[ A_z = -E t \]

\[ +\phi_p^{\text{in}}(t) \]

\[ \text{mixture of positive and negative frequency} \]

\[ A = 0 \]

\[ +\phi_p^{\text{in}}(t) = \frac{1}{\sqrt{2\omega_p}} e^{-i\omega t} \]

\[ \text{consequence of pair creation} \]

\[ \text{positive frequency} \]
Instantaneous particle picture

\[ A_z = -Et \]

\[ +\phi_p(t) \]

\[ \phi_p^{(t_0)}(t), -\phi_p^{(t_0)}(t) \]

a solution under the gauge \( A_z = -Et_0 \)

\[ +\phi_p(t) = \frac{1}{\sqrt{2\omega_p}} e^{-i\omega t} \]

positive frequency
Instantaneous particle picture

decompose $\phi^{\text{in}}_p(t)$ into positive and negative frequency instantaneously

$$+\phi^{\text{in}}_p(t) = \alpha_p(t_0) + \phi^{(t_0)}_p(t) + \beta^*_p(t_0) - \phi^{(t_0)}_p(t)$$

Field expansion

$$\phi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3p}{(2\pi)^3/2} \left[ +\phi^{\text{in}}(t)a^\text{in}_p + -\phi^{\text{in}}(t)b^{\text{in}\dagger}_p \right] e^{ip \cdot x}$$

$$= \frac{1}{(2\pi)^{3/2}} \int \frac{d^3p}{(2\pi)^3} \left[ +\phi(t_0)(t)a_p(t_0) + -\phi(t_0)(t)b^{\dagger}_p(t_0) \right] e^{ip \cdot x}$$

Instantaneous particle picture (creation and annihilation operator) is introduced.

particle pair distribution function

$$n_p(t) = \langle 0, \text{in} | a^\dagger_p(t) a_p(t) | 0, \text{in} \rangle \frac{(2\pi)^3}{V} = \langle 0, \text{in} | b^{\dagger}_p(t) b_p(t) | 0, \text{in} \rangle \frac{(2\pi)^3}{V}$$

$$= |\beta_p(t)|^2$$
Boson distribution

Particles are created first with about 0 momentum

Fermion distribution

accelerated according to classical eq. of motion

\[ p_z = e Et \]

\[ a = \frac{m^2}{2eE} = \frac{m^2 + p_T^2}{2eE} \]
Particles are created firstly with about 0 momentum.

Larger $a$, longer time to create first particles.

$n_p(t) \leq 1$ because of Pauli's exclusion principal.

accelerated according to classical eq. of motion

\[ p_z = eEt \]

\[
 a = \frac{m^2}{2eE} = \frac{m^2 + p_T^2}{2eE}
\]
**Back reaction** (numerical, 1+3 dim) Kluger et al. 1991

Couple the field eq. and Maxwell eq. \( \frac{dE}{dt} = -j \)

The time-evolution of the electric field

oscillating distribution in momentum space

Bose enhancement

Pauli blocking

\[ \alpha = 0.01 \] **boson**  
[fermion]
**Time scale of back reaction**

$t_V$ : the time the electric field strength becomes 0 at first

\[ t_V \gtrsim \sqrt{\frac{(2\pi)^3}{N_d g^3 E_0} e^{\pi m^2/2 g E_0}} \approx \frac{3 e^{\pi m^2/2 g E_0}}{N_d g^2 \cdot g E_0} \text{[fm]} \]

$N_d$ : a number of inner degrees of freedom

roughly estimated from the current without back reaction
**Time scale of back reaction**

\( t_V : \) the time the electric field strength becomes 0 at first

\[
t_V \approx \frac{3e^{\frac{\pi m^2}{2}gE_0}}{\sqrt{N_d g^2 \cdot gE_0}} \text{[fm]}
\]

\( m = 0 \)
\( gE_0 = Q_s^2 \approx 1 \text{GeV}^2 \)
\( t_V \approx \frac{3}{\sqrt{N_d g^2}} \text{[fm]} \)

\( N_d : \) a number of inner degrees of freedom

If \( g^2 \gtrsim O(1) \), \( t_V \) is the order or less than 1 fm.
Time scale of back reaction

\[ t_V : \text{the time the electric field strength becomes 0 at first} \]

\[
\begin{align*}
     t_V &> \sqrt{\frac{(2\pi)^3}{N_d g^3 E_0}} e^{\pi m^2/2 g E_0} \
     &\approx \frac{3e^{\pi m^2/2 g E_0}}{\sqrt{N_d g^2 \cdot g E_0} \text{ [GeV}^2]\}] \text{ [fm]}
\end{align*}
\]

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\[ N_d : \text{a number of inner degrees of freedom} \]

If \[ g^2 \geq O(1) \], \( t_V \) is the order or less than 1 fm.

Furthermore, a longitudinal magnetic field makes \( t_V \) smaller.
**Time scale of back reaction**

\( t_V \): the time the electric field strength becomes 0 at first

\[
t_V \gtrsim \sqrt{\frac{(2\pi)^3}{N_d g^3 E_0}} e^{\frac{\pi m^2 / 2 g E_0}{\sqrt{N_d g^2 \cdot g E_0}}} \approx \frac{3e^{\frac{\pi m^2 / 2 g E_0}{\sqrt{N_d g^2 \cdot g E_0}}}}{\sqrt{N_d g^2 \cdot g E_0}} \text{[fm]}
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\( N_d \): a number of inner degrees of freedom

**If** \( g^2 \gtrsim O(1) \), \( t_V \) is the order or less than 1 fm.

Furthermore, a longitudinal magnetic field makes \( t_V \) smaller.

This decay mechanism of an electric field plays an important role in the initial stage of heavy-ion collisions.
The effect of a magnetic field

Longitudinal magnetic field

\[ E = (0, 0, E), B = (0, 0, B) \]

- Landau level
  \[ p_T^2 = p_x^2 + p_y^2 \rightarrow (2n + 1)eB \quad (n = 0, 1, 2, \cdots) \]
- Zeeman effect
  \[ \pm 2seB \]

\[ m_T^2/eB \]

- Lowest level

\[ B = 0 \quad B \neq 0 \]

Continuous spectrum

Spin 0

Spin 1/2
The effect of a magnetic field

**Longitudinal magnetic field**

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  \[ \pm 2seB \]

The strong magnetic field makes particles “heavy” and suppresses pair creation.
The effect of a magnetic field

**Longitudinal magnetic field**

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- Landau level
  \[ p_T^2 = p_x^2 + p_y^2 \rightarrow (2n+1)eB \quad (n = 0,1,2,\cdots) \]
- Zeeman effect
  \[ \pm 2seB \]

\[ \frac{m_T^2}{eB} \]

- Continuous spectrum
- Spin 0
- Spin 1/2

The strong magnetic field makes particles “heavy” and suppresses pair creation.

The number of modes degenerating in one Landau level is proportional to B.

Pair creation is not suppressed.

Independent of B.
The effect of a magnetic field

Total particle number and current of fermions are enhanced by the magnetic field.

The time-evolution becomes faster due to the magnetic field.

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total particle number density (without back reaction)

time-evolution of the electric field (with back reaction)
Instability in spin-1 case

- Landau level: \( p_T^2 = p_x^2 + p_y^2 \rightarrow (2n+1) eB \) \( (n = 0,1,2,\ldots) \)
- Zeeman effect: \( \pm 2 s eB \)

\[ m_T^2 / eB \]

\( B = 0 \)
\( B \neq 0 \)

- lowest level

\[ eB \quad 0 \quad -eB \]

negative!!
Instability in spin-1 case

- Landau level \( p_T^2 = p_x^2 + p_y^2 \rightarrow (2n+1)eB \) \((n = 0, 1, 2, \cdots)\)
- Zeeman effect \( \pm 2seB \)

\[
2m_T/pzp + \omega = \exp(\mp i\omega_pt)
\]

Nielsen-Olesen instability
\[
\omega_p = \sqrt{p_z^2 + m_T^2}
\]
can be pure imaginary. \( \exp(\mp i\omega_pt) \) become unstable.
Summary

• We have revealed the momentum distribution of created particles and its time-evolution in uniform electric fields by defining particle picture instantaneously.
• The time scale of the decay process of the initial electric field due to back reaction has been estimated.
• We have shown that a magnetic field speeds up the time-evolution of the fermion system.

Remaining problems

• More realistic configuration of an electromagnetic field, especially the case that a field exists only inside the light-cone
• Interaction through quantum gauge field (collision)
• Instability in gluon case