Fluctuation and HBT results

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1. Approach and probes for critical phenomena relevant to QCD phase transitions
2. HBT
3. Two particle correlation via differential analysis on multiplicity fluctuations
4. Summary

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Approach to QCD phase diagram at RHIC

- High opacity state
- Bulk matter flow with quark d.o.f
- High temperature state

We can approach from well above $T_c$

1st order ?

Deconfinement p.t. (?)

Crossover $m_q 
eq 0$

CP ?

$T=221 \pm 23$ (stat) $\pm 18$ (sys)

Lattice result $T_c \sim 170$ MeV

High opacity state
- Bulk matter flow with quark d.o.f
- High temperature state

Chiral p.t.
Experimental approaches to critical phenomena

• \(\langle qq\rangle\):
• \(J/\psi\) suppression (deconfinment)
• low mass vector mesons and dilepton continuum (chiral)

• **Bulk collective observables:**
  • Isothermal compressibility: Multiplicity fluctuations
  • Heat capacity: Mean \(pT\) fluctuations
  • Particle ratio and fluctuations on particle compositions
  ➢ Duration time of particle emission: HBT
  ➢ Correlation length and the strength:
    • density-density correlation in longitudinal space
  • Sound velocity via eccentricity scaling of \(v2\)
  • Viscosity to entropy ratio with \(v2\) and \(R_{AA}\)
HBT as a probe of 1st order P.T.

\[ C_2(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)} = \frac{\text{real event pairs}}{\text{mixed event pairs}} \]

\[ s(T) = \left( \frac{T}{T_c} \right)^3 \left( 1 + \frac{dQ}{dQ + dH} \tanh \left( \frac{T - T_c}{\Delta T} \right) \right) \]

where \( s_c = \text{const.} \times \frac{1}{2}(d_Q + d_H)T_c^3 \) is the entropy density at \( T_c \).
3-D imaging of source shape

\[ C(q) - 1 = R(q) = \int dr K(q,r) S(r) \]

\[ R(q) = \sum_{l, \alpha_1, \ldots, \alpha_l} R^l_{\alpha_1, \ldots, \alpha_l}(q) A^l_{\alpha_1, \ldots, \alpha_l}(\Omega_q) \]

\[ S(r) = \sum_l \sum_{\alpha_1, \ldots, \alpha_l} S^l_{\alpha_1, \ldots, \alpha_l}(r) A^l_{\alpha_1, \ldots, \alpha_l}(\Omega_r) \]

Correlation moment

Encoding

FS interaction

Source function

Cartesian surface-spherical harmonic basis

\[ A^l_{\alpha_1 \ldots \alpha_l} \]

\( l \): index from 0 to 6 here

\( \alpha \): spatial index

\[ R^l_{\alpha_1, \ldots, \alpha_l}(q) = 4\pi \int dr r^2 K(q,r) S^l_{\alpha_1, \ldots, \alpha_l}(r) \]

Inversion from \( R^l(q) \) to \( S^l(r) \)

No assumption on source shape

In PCMS

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3-D imaging results

Therminator Model

- Bjorken type longitudinal boost invariance
- Blast-wave transverse expansion
- Thermal emission from a longitudinal cylinder

\[ t^2 = (\tau_0 + a\rho)^2 + z^2 \]

\( \tau_0 \) is proper breakup time at \( \rho = 0 \)

\( \tau = \tau_0 + a\rho \) is proper breakup time at \( \rho \) with \( a = -0.5 \) (burn outside in)

\( \tau \) is replaced by \( \tau' \) with probability of \( \frac{dN}{d\tau'} = \theta(\tau' - \tau) / \Delta\tau \exp\left[-(\tau' - \tau) / \Delta\tau\right] \)

Mean proper emission time

\( \Delta\tau = 2\text{fm/c} \) is small but finite! 

\( \tau_0 \sim 9\text{fm/c} < |\Delta t_{\text{LCM}}| < \sim 12\text{fm/c} \)
Low pT two particle correlation in STAR

Analyzed 1.2M minbias 200 GeV Au+Au events, and 13M 62 GeV minbias events (not shown) Included all tracks with $p_T > 0.15$ GeV/c, $|\eta| < 1$, full $\phi$

We see the evolution of correlation structures from peripheral to central Au+Au

Slide from M. Daugherity, STAR Collaboration presented at QM08

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Fit Function (in 5 Easy Pieces)

Proton-Proton fit function

$\rho(\Delta \eta, \Delta \phi) = \frac{\rho(\Delta \eta, \Delta \phi)}{\rho_{\text{ref}}} + \frac{\rho_{\text{ref}}}{\sqrt{\rho_{\text{ref}}}}$

STAR Preliminary

HBT, res., e+e-
2D exponential

Minijet Peak
2D gaussian

Away-side
-cos($\phi$)

longitudinal fragmentation
1D gaussian

dipole

quadrupole

$\cos(2 \phi_{\Delta})$

Dipole term

Use proton-proton fit function + $\cos(2 \phi_{\Delta})$ quadrupole term ("flow"). This gives the simplest possible way to describe Au+Au data.

Note: from this point on we’ll include entire momentum range instead of using soft/hard cuts

Au-Au fit function

M. Daugherity, STAR Collaboration
Does the transition from narrow to broad $\eta_\Delta$ occur quickly or slowly?

Shape changes little from peripheral to the transition

Large change within ~10% centrality

Smaller change from transition to most central

Low-$p_T$ manifestation of the “ridge”

The transition occurs quickly

Slide from M. Daugherity, STAR Collaboration presented at QM08
A picture of expanding medium in early stage

Initial stage

External field $h$

$T > T_c$

Longitudinal field density fluctuations from the mean density is a natural order parameter

$\phi(z) = \rho(z) - \langle \rho \rangle$

Many modes from small to large appear at $T = T_c$. A typical length scale disappears i.e. transition to power law

$T < T_c$

We may expect freeze of initially embedded fluctuation due to rapid dilution of medium in the longitudinal direction
Direct observable for Tc determination

GL free energy density $g$ with $\phi \sim 0$ from high temperature side is insensitive to transition order, but it can be sensitive to Tc

$$g(T, \phi, h) = g_0 - \frac{1}{2} A(T)(\nabla \phi)^2 + \frac{1}{2} a(T)\phi^2 + \frac{1}{4} b\phi^4 + \frac{1}{6} c\phi^6 \cdots - h\phi$$

Fourier analysis on

$$G_2(y) = \langle \phi(0)\phi(y) \rangle$$

$$\langle |\phi_k|^2 \rangle = Y \int G_2(y)e^{-iky}dy$$

$$\langle |\phi_k|^2 \rangle = \frac{NT}{Y} \frac{1}{a(T) + A(T)k^2}$$

1-D two point correlation function

$$G_2(y) = \frac{NT}{2Y^2 A(T)} \xi(T)e^{-|y|/\xi(T)}$$

Correlation length

$$\xi(T)^2 \equiv \frac{A(T)}{a_0(T - T_c)}$$

Susceptibility

$$\chi_k = \frac{\partial \phi_k}{\partial h} \propto \left( \frac{\partial^2 (g - g_0)}{\partial \phi^2_k} \right)^{-1} = \frac{1}{a_0(T - T_c)(1 + k^2 \xi^2)}$$

Susceptibility in long wavelength limit

$$\chi_{k=0} = \frac{1}{a_0(T - T_c)} \propto \frac{\xi}{T} G_2(0)$$

Product between correlation length and amplitude can also be a good indicator for T~Tc
Density measurement: $dN_{ch}/d\eta$

Negative Binomial Distribution (NBD) perfectly describes multiplicities in all collision systems and centralities at RHIC.
Differential multiplicity fluctuations

$\Delta \eta$ window

$\Delta \eta < 0.7$ integrated over $\Delta \phi < \pi/2$ and $pT>0.1\text{GeV}$

Zero magnetic field to enhance low pt statistics per collision event.
NBD fits at each window size in CuCu@200

16 fit examples in most left edge in top 10% events out of \(2^8/2^*(1+2^8)\) times NBD fits

Level (window size)

\[L = 2^8 (1-\delta \eta/\Delta\eta_{PHENIX})\]
Two point correlation via NBD

\[ P_n^{(k)} = \frac{\Gamma(n+k)}{\Gamma(n-1)\Gamma(k)} \left( \frac{\mu/k}{1+\mu/k} \right)^n \left( \frac{1}{1+\mu/k} \right)^k \]

\[ \sigma^2 = \frac{1}{\mu} + \frac{1}{k} \quad \mu \equiv <n> \]

1/k corresponds to integral of two point correlation

source 1

source 2

source 1+2

Uncorrelated sources

Correlated sources

k=1 Bose-Einstein

k=\infty Poisson

k=k_1

k=k_2

k=k_1+k_2
Extraction of $\chi_{k=0}^*T$

**Fit with approximated functional form**

- **a)** 10%
- **b)** 5%

Approximated functional form

$$k(\delta\eta) = \frac{1}{2\alpha\xi / \delta\eta + \beta} \quad (\xi \ll \delta\eta)$$

Look at slopes

Parametrization of two particle correlation

$$C_2(\eta_1, \eta_2) \equiv \rho_2(\eta_1, \eta_2) - \rho_1(\eta_1)\rho_1(\eta_2)$$

$$\frac{\rho_1^2}{\rho_1^2} = \alpha e^{-\delta\eta/\xi} + \beta$$

$\beta$ absorbs rapidity independent bias: Npart fluctuation and reaction plane rotation and $v_2$

Exact relation with NBD $k$

$$k^{-1}(\delta\eta) = \frac{\langle n(n-1) \rangle}{\langle n \rangle^2} - 1$$

$$= \int_0^{\delta\eta} \int_0^{\delta\eta} C_2(\eta_1, \eta_2) d\eta_1 d\eta_2$$

$$= \frac{2\alpha\xi^2 (\delta\eta / \xi - 1 + e^{-\delta\eta/\xi})}{\delta\eta^2} + \beta$$


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Correlation functions and correlation length

Used in E802

\[ C_2 = 1 + R(0,0)e^{-|y_1 - y_2|/\xi} \]

\[ k(\delta \eta) = \frac{1}{R_0} \frac{\delta \eta / 2\xi}{[1 - (\xi / \delta \eta)(1 - e^{-\delta \eta / \xi})]} \]

General correlation function

\[ C_2 = 1 + \frac{R_0}{|y_1 - y_2|^\alpha} e^{-|y_1 - y_2|/\xi} \]

\[ k(\delta \eta) = \frac{\delta \eta}{\int_{\delta \eta}^{\infty} \frac{R_0}{y^\alpha} e^{-y/\xi} dy} \]

\( \xi \) : correlation length, \( \alpha \) : critical exponent

Using arbitrary \( R_0, \xi \) and \( \alpha \).

One may discuss an effective potential form.
\( \alpha \xi, \beta \) vs. \( N_{\text{part}} \)

Dominantly \( N_{\text{part}} \) fluctuations and possibly correlation in azimuth

\( \beta \) is systematically shift to lower values as the centrality bin width becomes smaller from 10\% to 5\%. This is understood as fluctuations of \( N_{\text{part}} \) for given bin widths.

\( \alpha \xi \) product, which is monotonically related with \( \chi_{k=0} \) indicates the non-monotonic behavior around \( N_{\text{part}} \sim 90 \).

\[
\alpha \xi = \chi_{k=0} T / \bar{\rho}_1^2 \propto \bar{\rho}_1^{-2} \frac{T}{|T - T_C|}
\]

Significance with Power + Gaussian:
3.98 \( \sigma \) (5\%), 3.21 \( \sigma \) (10\%)

Significance with Line + Gaussian:
1.24 \( \sigma \) (5\%), 1.69 \( \sigma \) (10\%)
Comparison of three collision systems

- Cu+Cu@200GeV
- Au+Au@200GeV
- Au+Au@62.4GeV

Comparison of three collision systems

- Normalized mean multiplicity to that of top 5% in Au+Au@200GeV
- N_{part} \sim 90 in Au+Au@200GeV
- \varepsilon_{BJ} \tau \sim 2.4 \text{GeV}/\text{fm}^2/\text{c}

Similarity to STAR mini jet results at low $p_T$

Equivalent quantity;

$$\chi_T \propto \alpha \xi \mu^2 \propto \text{amplitude x width}$$

shows similar trends to what STAR sees.

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In lower $K_{\text{ET}}$, there seems to be different behaviors between baryon and mesons. The transition is at $N_{\text{part}} \sim 90$.

Low mass sigma field may repulse pions and attract protons according to hep-ph/0504048 by E. Shuryak. Can this phenomenon be understood as such effects?
How about $<c\bar{c}>$ suppression?

**Au+Au@200GeV**

$N_{\text{part}} \approx 90$ in AuAu@200GeV

$\varepsilon_{BJ\tau} \approx 2.4$ GeV/fm²/c

**Cu+Cu@200GeV**

**J/$\psi$ suppression pattern**

$R_{AA}$

$|y| < 0.35$

$N_{\text{part}}$

arXiv:0801.0220v1 [nucl-ex]

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Summary

1. RHIC created strongly coupled high temperature & opaque state with partonic d.o.f. This is the very beginning of the scientific program on quantitative understanding of the QCD phase structure.

2. 3D imaging of source shape from HBT type correlation suggests small but finite duration of pion emission time. There seems to be no strong indication of 1st order P.T. at 200GeV, but worth measuring at lower energy (in higher baryon density).

3. PHENIX and STAR see very similar rapid transition of the two particle correlation in longitudinal direction at the similar centrality range in Au+Au@200 and 62GeV. However, a caution is necessary, because the rapidity window size is totally different: PHENIX is limited within 0.7 and perhaps STAR misses information at short rapidity window by the brute force subtraction process. Nevertheless, it is interesting to foresee the common reasoning.
Open issue

- Is the rapid transition related with creation of CGC flux tube?
- Is the color electric field in tubes related with bag pressure (confinement)?
- Where is the threshold of the creation of the thermal system?

**Figure from K. Itakura at ISMD08**

A slide from S.H. Lee’s talk on 14 Oct

QCD vacuum \(< \frac{\alpha}{\pi} G^2 >_0 = (0.35 \text{ GeV})^4\)  
Large increase in \(E^2\)

Vacuum with negative pressure

\(< \frac{\alpha}{\pi} G^2 >_T = 0.7 \times < \frac{\alpha}{\pi} G^2 >_0\)

Centrality can scan \(A\)

\(Q_s \propto A^{1/6} \sqrt{S} \sim 0.3 / 2\)